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ABSTRACT

This is one of a series of geometry modules developed for use by secondary students in a laboratory setting. This module, intended to present a treatment of congruent triangles which is not totally axiomatic, contains six sections: (1) Draw vs. Construct: (2) Triangle Construction: (3) Arguing for Congruence: (4) Parallel Line Construction: (5) Altitudes, Medians, and Angle Bisectors: and (6) Triangle Inequalities. The module includes compass and straight-edge constructions as a tool in the investigation of triangles, which allows for the de-emphasis of the deductive system by making use of inductively drawing conclusions. The module includes teaching suggestions and activities for each section. (MK)

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GEOMETRY MODULE FOR USE

IN A

MATHEMATICS LABORATORY SETTING

CONGRUENCE AND CONSTRUCTION

by

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SE 031 121

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Congruence and Construction

Teacher Guide

Objectives

1. The student will be able to define and recognize congruent triangles.
2. The student will be able to present an acceptable argument for the congruence of two triangles.
3. The student will be able to perform the following constructions: the duplication of a segment, angle and triangle, the bisection of a segment and angle, the construction of a perpendicular and a parallel to a given line.
4. The student will be able to recognize and construct medians, altitudes and angle bisectors of triangles.
5. The student will know and be able to use single triangle inequalities.

Overview

In one widely used geometry text a student may be asked to complete up to 135 proofs involving congruent triangles in one particular chapter. He manages to fight his way through these, reads the title of the next chapter which is "A Closer Look at Proof" and collapses. We believe that it is generally in a treatment of congruent triangles that a total axiomatic development of Euclidean Geometry becomes unbearable for most students. Therefore two major changes are made in this module.

The most visible of these is the inclusion of straight edge and compass construction as a tool in the investigation of congruent triangles. This opens the way for the most signigicant change, the de-emphasis of the deductive system. Instead, much use is made of inductively drawing conclusions. In doing what is normally refered

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to as proof, we are concerned only that the student can present a convincing (to the teacher) argument.

The module contains six basic sections, some of which are subdivided into various activities. They include:

I. Draw vs. Construct

II. Triangle Construction

SSS, Definition of congruent triangles, SAS, ASA and AAS, SSA and AAA, HL

III. Arguing for Congruence

IV. Parallel lines Construction

V. Altitudes, Medians and Angle Bisectors

VI. Triangle Inequalities

The first of these not only indicates the difference between drawing and construction, but with assistance from the instructor indicates why we chose to use constructions rather than drawing in this development.

The second section contains the central ideas of the module. The student is given the definition of congruent triangles and the methods of determining congruence between triangles. The major tool of this section is the construction activity. Several triangles are constructed by a given method which leads to an inductive conclusion about the value of that method in determining congruence.

The conclusions reached here are put to work in the third section as students are asked to prove congruence between triangles. The word proof is used here to avoid confusion and not to indicate deductive rigor. It is intended that each teacher can demand whatever level of completeness he judges his students can handle.

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The fourth and fifth sections treat constructions which do not vary greatly from most standard textbook treatments. The section on triangle inequalities is included here since it is felt to contain necessary information for a complete geometry course and this was a convenient spot to include it.

Materials

Compass

Ruler

Protractor

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Draw vs Construct

Teachers Guide

Teaching Suggestions

1. The first activity is designed to have the students work on it with no preparation. The students should not be asked to bring in tools for this activity.
2. The activities are best attempted on an individual basis.
3. The instructor should make the distinction between drawing and cnstruction. Notable differences are: drawing allows the use of any instrument while construction allows only straight edge and compass, drawing allows the use of numbers to represent measures and the converse and construction does not.
4. There are three general reasons for choosing constructions rather then drawing in this module.
   a) Construction demands a familiarity with numerous geometric properties of figures while drawing does not require it. Construction is therefore better from an instructional point of view.
   b) Construction avoids the inaccuracy of estimating a numerical correspondent to a distance or angle measure and then converting that back to a set of points.
   c) One may not always have a protractor and a ruler. However a straight edge is always available. A compass is easily devised.
5. An additional activity you may wish to include is the making of compasses. You might provide some useful items to work with, old computer cards for example.
6. No solutions are given for the constructions in this activity. It is useful

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to promote a class discussion about what will work and what will not work.

7. Answers to questions in the activity are:

   4. AP = BP = CP

   5. On the circle with center P and radius AP.

   6. No

   7. Yes

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Triangle Construction

Teacher Guide

Teaching Suggestions

1. This unit includes an activity for each of the triangle constructions SSS, SAS, AAS, HL; an activity to define congruent triangles, and two activities to explore construction by SSA and AAA arrive at conclusions.
2. Each activity is followed by exercises that require no special equipment and therefore do not have to be done in class.
3. The activities are all individual except the second (defining congruent triangles) which is done in groups as indicated in the activity using chairs to fill out the groups.
4. Activities can be checked by a transparency of the construction displayed on the overhead or passed around the room to use as an overlay.
5. Throughout the activities are questions that ask the student to supply a reason or conclusion. Discuss these with the class. There are also important points included as exercises that require teacher emphasis. Be sure ways of proving triangles congruent are understood before doing the next activity.
6. In these activities students are asked to draw conclusions about their constructions. Keep the class reminded of the dangers of drawing conclusions from too few cases, challenge their conclusions, encourage critical thinking.
7. Throughout the activities are questions to lead students through an idea. Require students to record an answer for each question.

Materials

1. Compasses
2. Rulers
3. Protractors

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Exercise Answers - SSS (C-6)

1. Triangle 1 - scalene
   Triangle 2 - isosceles
   Triangle 3 - equilateral
   Triangle 4 - none

2. Triangle 1 - right
   Triangle 2 - acute
   Triangle 3 - acute
   Triangle 4 - none

3. When the sum of the lengths of two sides is not greater than the third.

4. Three angles are congruent
   Two angles are congruent
   No angles congruent

Exercise Answers - Definition of congruent triangles (C-12)

1. a. T
   b. T
   c. F
   d. T

2. a. $\triangle RSV \cong \triangle TSV$
   b. $\triangle WYD \cong \triangle XEZ$
   c. $\triangle ADC \cong \triangle CBA$
   d. $\triangle GIJ \cong \triangle HJI$

3. $\triangle ABC \cong \triangle ONM$
   $\triangle DEF \cong \triangle HIJ$
   $\triangle JKL \cong \triangle ORP$

4. a. $\triangle AXR \cong \triangle SYR$
      $\triangle AXB \cong \triangle SYR$
      $\triangle ARB \cong \triangle SBR$
   b. $\triangle CEB \cong \triangle CAD$
      $\triangle BAE \cong \triangle DEA$
      $\triangle AOB \cong \triangle EOD$

5. $\triangle ABC \cong \triangle NMO$
   $\triangle ABC \cong \triangle OMN$

6. Congruence is reflexive, symmetric, transitive.

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Triangle Construction

Teacher Guide

Exercise Answers - SAS (C-19)

1. Second triangle
2. First triangle
3. None
4. First triangle
5. Second triangle
6. None
7. Answers will vary
8. Yes

Exercise Answers - ASA (C-21)

1. Yes
2. Scalene, acute
3. a. SSS $\triangle XYZ$
   b. ASA $\triangle AC?$
   c. SAS $\triangle DEF$
   d. SAS $\triangle GHI$
   e. ASA $\triangle WX?$
   f. SSS $\triangle AXC$

Exercise Answers - AAS (C-26)

1. a. SSS
   b. ASA
   c. ASA
   d. SAS
   e. AAS
   f. ASA
2. Probably students will name SSA and AAA.
3. Yes.

Exercise Answers - SSA (C-29)

1. Not necessarily! Discuss this.
2. 3 cm

   $m < 3$ cm

   $xy > m > 3$ cm

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Exercise Answers - AAA (C-31)

1. No. their total measure must be 180°.
2. Yes
3. Not necessarily
4. AAS
5. a. SSS
   b. AAS
   c, ASA
   d. ASA
   e. SAS
   f. AAS
   g. ASA
6. Yes
7. SSS, $\triangle ADC \cong \triangle CBD$, Def. congruent triangles
8. ASA, $\triangle BCD \cong \triangle DAB$, Def. congruent triangles
9. $\triangle ACE \cong \triangle BCD$, SAS (others possible)
10. ASA

Exercise Answers - HL (C-40)

1. AAS
   SAS
   AAS
   HL
2. a. ASA
   b. SAS
   c. HL
   d. ASA
   e. HL
   f. SSS

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Altitudes, Medians and Angle Bisectors

Teacher Guide

Teaching Suggestion

1. Have the students bring compasses and straight edges to class. This section should just about teach itself.

Materials

Compass	one per student
Straight edge	one per student

Answers to Questions

1. Yes
2. They are equal
3. Yes
4. Each pair has ratio 1:2
5. No
6. Yes
7. $\triangle FUZ \cong \triangle DUZ$ by SSS, $\angle FUZ \cong$ DUZ
8. Yes
9. They are equal

Parallel Lines Construction

Teacher Guide

Teaching Suggestions

Both the activities and exercises are constructions. Completed work can be checked against a transparency on the overhead or a transparency used as an overlay.

Materials

1. Compass
2. Straightedge

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Arguing for Congruence

Teachers Guide

Teaching Suggestions

1. Very little direction has been given to the student in this section. Since most geometry teachers have taught this repeatedly and have developed their own methods for doing so, we leave it to the teacher to provide more specific directions.
2. Two column proofs are not exhibited here since stating reasons requires a complete reference on past theorems. Since not all theorems in this course are specifically stated as such two column proofs would be much more difficult to develop.
3. The authors feel strongly that the aim of this section should not be the production by students of rigorous formal proofs. Instead we wish to have the student argue convincingly in writing for the validity of a statement.
4. Use your imagination in pursuing the section. For example, a possible approach is to divide the class into small groups, assign each group an exercise and have each group develop an argument for that exercise. Each group must in turn try to convince the class of the validity of their argument.

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Triangle Inequalities

Teacher Guide

Teaching Suggestions

1. This activity contains a proof where reasons must be supplied. Some students will need teacher help with this.
2. Students are asked to form triangles of two paper strips and a ruler. Anything of convenient length could be substituted here (sticks for example).

Materials

1. Rulers
2. Two paper strips of different lengths for each person or each small group.

Exercise Answers

1. $\overline{WV}$
2. LC
   LB
   LE
   Smallest angle opposite shortest side, etc.
3. The perpendicular from the point to the line.
4. b
5. The distance from a point to a line is the length of the segment of the perpendicular from the point to its intersection with the line.
6. 9, 5
7. $x + y$, $x - y$
8. a. Yes
   b. No
   c. Yes
   d. Yes
   e. No
   f. Yes
   g. Yes
   h. No

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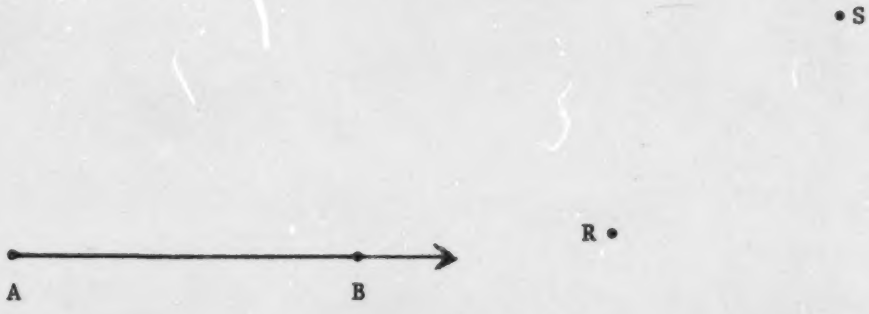
Draw vs. Construct

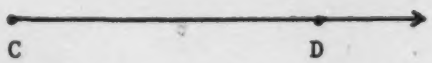
Activity I

Using only whatever items you have with you, complete the following activities in the best way you can.

Part I On ray $\overrightarrow{AB}$ mark off a segment $\overline{AX}$ such that AX = RS.



Part II Using C as a vertex and $\overrightarrow{CD}$ as one of the sides, draw an angle $\angle ECD$ such that $m\angle ECD = 60$.



Questions

1. List all the tools used in making the drawing.
2. How accurate do you believe your drawings are? (Example $\pm 2^{\circ}$, $\pm$ 1 mm)
3. To make your particular drawings was it necessary to be aware of any "facts of geometry"? If so, what facts?

In the previous activities one of two basic methods could apply in accomplishing these tasks. One could draw the required figures or one could construct them.

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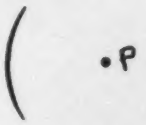
Your instructor will explain to you the difference between drawing and construction, which we will use in this course, and why we will use it.

Now armed with your construction tools, a straight edge and compass, there are a few basic ideas of construction that you should know.

We shall call the curve made by a compass an arc with its center at the compass point P.



Part III With your compass, construct an arc with center P. Locate three points A, B and C on that arc.

Questions 4. How do lengths AP, BP and CP compare?

5. Where would you find a point X such that PX = AP?

6. We shall call distance AP the radius of the arc. Does the circle with center P and radius AP differ from the set of all points at a distance of AP from P?

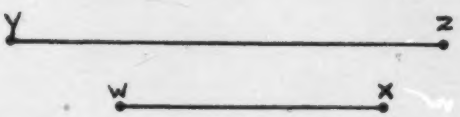
Use your compass to repeat Part I.

Question 7: Is X the only point on $\overrightarrow{AB}$ which is also distance RS from A?

You can now duplicate any given segment.

Exercises

You are given segments $\overline{WX}$ and $\overline{YZ}$ of length 3 orbs and 5 orbs respectively.



1. Construct a segment the length of 5 orbs.
2. Construct a segment of length 8 orbs.
3. Construct a segment of length 1 orb.
4. Construct a segment of length 11 orbs.

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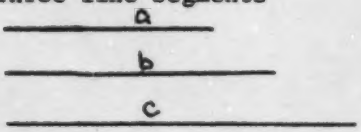
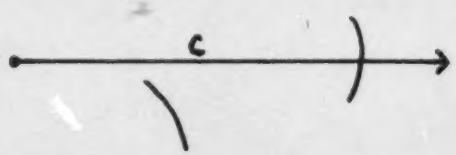
Triangle Construction - SSS

Activity II

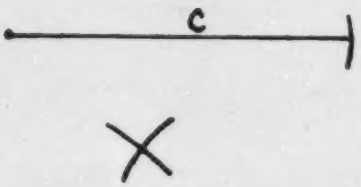
Given: Three line segments



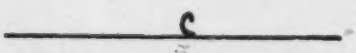
Construct: A triangle with the three line segments as sides.



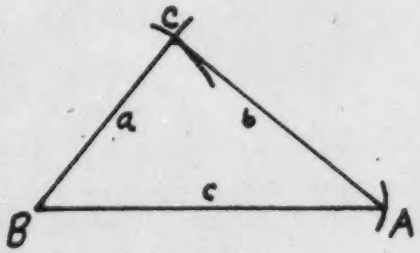
1. Duplicate one line segment c.



2. With radius equal to measure of a and center at one end point of c construct an arc.



3. With radius equal to measure of b and center at other endpoint of c construct another arc that intersects the first.



4. Draw from intersection of arcs to endpoints of c. Label vertices of the triangle.

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1. Given: Three line segments

   3 cm ———————

   4 cm ———————————

   5 cm ———————————————

   Construct: A triangle having the given line segments as sides

2. Given: Two line segments 5 cm long and one segment 3 cm long. (Use a ruler to draw them here.)

   Construct: A triangle having the given line segments as sides.

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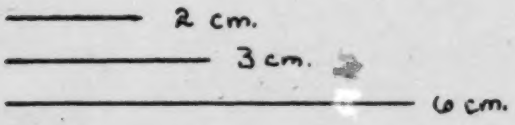
3. Given: A line segment. (Choose any length you like, use a ruler to draw it here.)

Construct: A triangle having three sides congruent to the given line segment.

4. Given: Three line segments.



Construct: A triangle having the given line segments as sides.

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Triangle Construction - SSS

Exercises

1. Determine whether each of the triangles you constructed is scalene, isosceles or equilateral.

2. Determine whether each of the triangles you constructed is acute, obtuse, or right.

3. Under what conditions is it impossible to construct a triangle from three given line segments?

4. Use a protractor to measure and record the measures of the angles of each triangle you constructed. What do you conclude about the angles of this equilateral triangle? What do you conclude about the angles of this isosceles triangle? What do you conclude about the angles of this scalene triangle?

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Definition of Congruent Triangles

Activity III

Your teacher will arrange the class in groups of six members each - 3 boys and 3 girls. If your group is short a member create a "chair person" to fill in. Choose a name of the proper gender, write it on a piece of paper, and put it on the chair so you can refer to the "chair person" by name.

The "Correspon - DANCE"

1. For the first round let each boy choose a girl to be his partner. Record the couples by name.

   Couple 1

   Couple 2

   Couple 3

   The pairing is called a correspondence. Each member of the set of boys has selected a member of the set of girls and each member of the set of girls has been selected.

2. The second round is ladies choice. The girls must choose partners and find a <u>different</u> correspondence. Record the couples.

3. Continue finding correspondences until you've found as many <u>different</u> correspondences as you can. Record each.

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Let the vertices of triangle JOY be one set and the vertices of triangle SAD be another set. Indicating a pairing by "$\longleftrightarrow$" one correspondence might be

$J \longleftrightarrow S$

$O \longleftrightarrow D$

$Y \longleftrightarrow A$

For convenience we write $JOY \longleftrightarrow SDA$ where the correspondence is indicated by the order of the letters. Note that $YOJ \longleftrightarrow ADS$ is the same correspondence. Make a list of all the different correspondences between the vertices of ΔJOY and ΔSAD.

Suppose we have the following correspondence between two triangles: $XYZ \longleftrightarrow DEF$. Since $X \longleftrightarrow D$ and $Y \longleftrightarrow E$ we say $\overline{XY} \longleftrightarrow \overline{DE}$. They are corresponding sides of the two triangles. Below are listed the three sides and three angles of ΔXYZ. Supply the part from ΔDEF that corresponds to it.

1. $\overline{XY} \longleftrightarrow$
2. $\overline{YZ} \longleftrightarrow$
3. $\overline{ZX} \longleftrightarrow$
4. $\angle X \longleftrightarrow$
5. $\angle Y \longleftrightarrow$
6. $\angle Z \longleftrightarrow$

If each of the six correspondences listed above is also a congruence we say the triangles are congruents.

Definition: Two triangles are congruent if and only if there exists a correspondence between them such that each part of one triangle is congruent to the corresponding part of the other triangle.

$\Delta XYZ \cong \Delta DEF$

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Name _____

Given: Three line segments, lengths 6 cm, 7 cm, 9 cm. (Draw the given here.

Construct: A triangle having these line segments as sides. (Be very careful on this construction.)

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Put your name on your triangle construction to insure getting it back. Use ruler and protractor to measure the six parts of your triangle and record the results. Trade with others in the room and do the same until you've completed the table. Measure carefully.

Name	Short Side	Second Side	Longest Side	Smallest Angle	Second Angle	Largest Angle
1. Yours						
2.						
3.						
4.						
5.						
6.						
7.						
8.						

Conclusion: All the triangles were constructed from the same three given sides. Checking all six parts we find all the triangles are _____.

If three sides of one triangle are congruent respectively to three sides of another, the triangles are congruent. (We abbreviate this conclusion to simply SSS.)

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You arrived at this important conclusion by investigating many cases and generalizing what you observed. This method of reasoning is called inductive reasoning and we shall use it often in this study of congruent triangles. However, like the blind boy who felt an elephants trunk and decided an elephant is like a snake you ought to be very careful about arriving at conclusions too soon.

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Congruent Triangles

Exercises

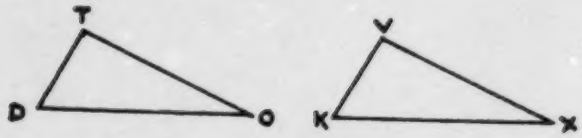
1. Basing your decision on appearance, tell whether each statement is true or false.

   a) $\Delta DOT \cong \Delta DXV$

   b) $\Delta VKX \cong \Delta TDO$

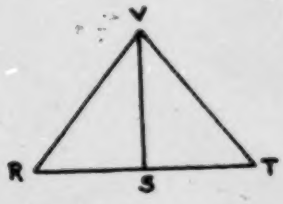
   c) $\Delta KXV \cong \Delta TDO$

   d) $\Delta TDO \cong \Delta TDO$

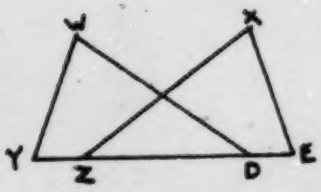


2. In each of these drawings there are two congruent triangles. Write correspondences that describe the congruences.

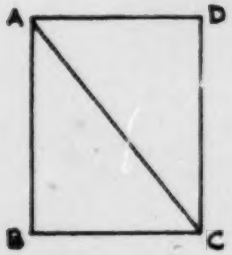
   a)



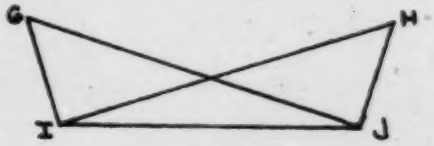
   b)



   c)

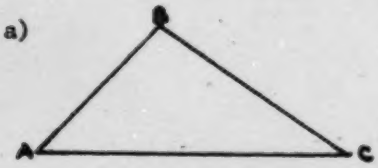


   d)



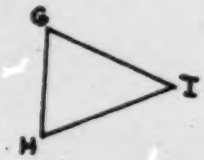
3. Assume that figures which appear to be congruent are congruent. List all correspondences that describe congruences.

   a)



   b)

   c)



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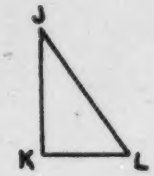
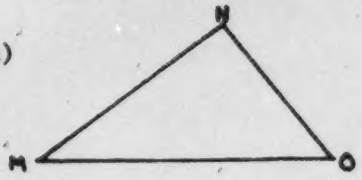
d)



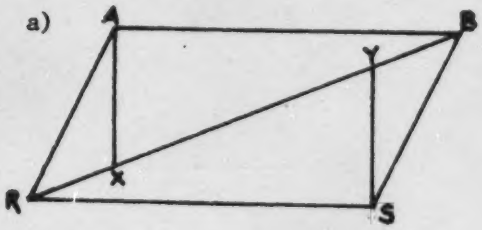
e)



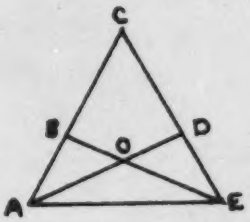
f)

4. Assume that triangles which appear to be congruent are congruent. List all correspondences that describe congruences.

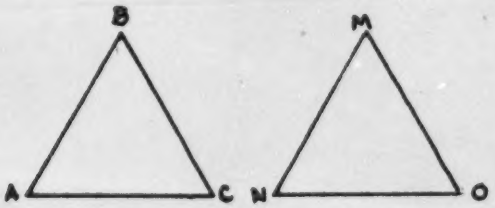
a)



b)



5. ΔABC and ΔMNO are isosceles triangles with $\overline{AB} \cong \overline{BC} \cong \overline{MN} \cong \overline{MO}$ and $\overline{AC} \cong \overline{NO}$



List the possible correspondences that you think might be congruences.

6. Is congruence of triangles reflexive, symmetric, or transitive?

7. Write a definition of correspondence.

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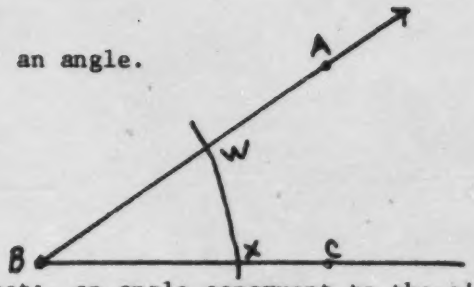
Triangle Construction - SAS

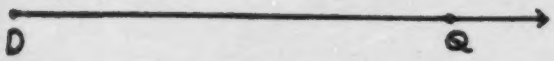
Activity IV

Given: an angle.

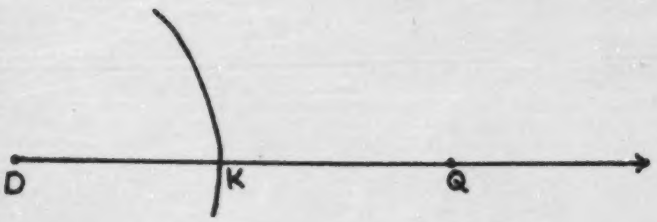


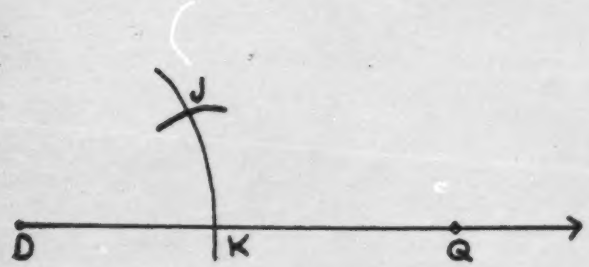
Construct: an angle congruent to the given angle.

1. The angle can be constructed with any given ray as its side.

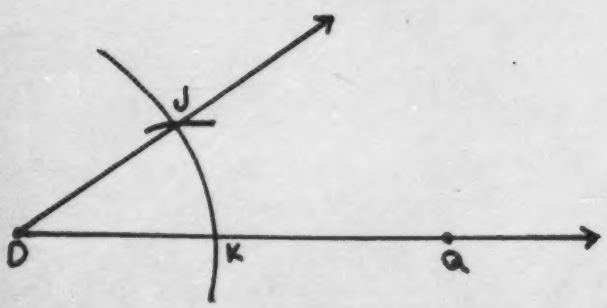


2. With any convenient radius and center at B construct an arc intersecting $\overrightarrow{BA}$ at W and $\overrightarrow{BC}$ at X. Use the same radius to construct an arc with center at D. Call the point K where the arc intersects $\overrightarrow{DQ}$.





3. With radius WX and center K construct an arc. Call the intersection of the arcs J.



4. Draw $\overrightarrow{DJ}$.

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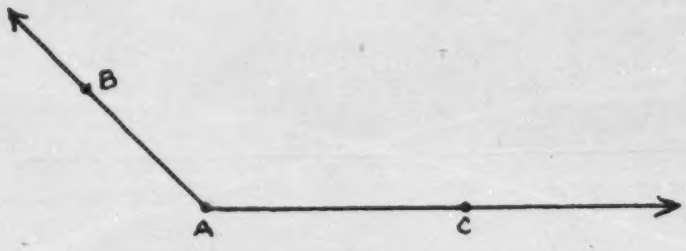
How do you know $\angle ABC \cong \angle JDK$?

$\overline{BW} \cong \overline{DJ}$, $\overline{BX} \cong \overline{DK}$, $\overline{WX} \cong \overline{JK}$ by construction

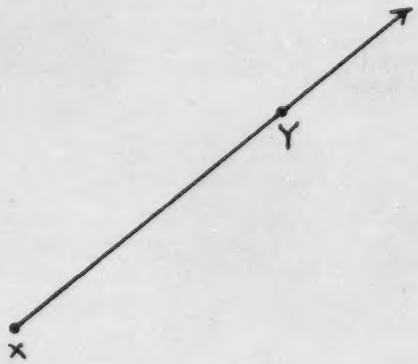
$\Delta ABC \cong \Delta JDK$ SSS postulate

$\angle ABC \cong \angle JDK$ Definition of congruent triangles.

1. Given: an angle



Construct: an angle congruent to $\angle BAC$ with $\overrightarrow{XY}$ as a side.



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2. Given: $\angle A$ any acute angle, $m\angle B = 20^{\circ}$. (Draw the given.)

Construct: $\angle C$ such that $m\angle C = m\angle A + m\angle B$.

3. Given: $\angle x$ any obtuse angle, $m\angle Y = 30^{\circ}$. (Draw the given.)

Construct: $\angle z$ such that $m\angle z = m\angle x - m\angle Y$.

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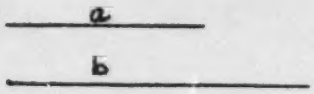
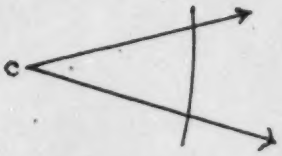
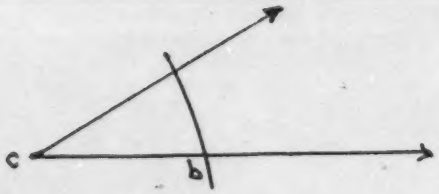
4. Given: two sides of a triangle and the included angle (that is the angle between them or formed by them).





Construct: the triangle



1. Construct ∠C with vertex at endpoint of one given segment.
2. The rest is left to you. Where would you construct side a so that a and b form angle C?
3. Draw a line segment from end point of a to endpoint of b. Label the vertices.

Compare this triangle with someone else's constructed from the same given. Conclusion?

5. Given: $M\angle A = 105^{\circ}$, AB = 4.5 cm, AC = 6 cm (use protractor and ruler to draw the given here).

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Construct: ΔABC

6. Given: $AB = AC = 5.5$ cm, $m\angle A = 20^\circ$ (Draw the given).

Construct: ΔABC

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Triangle Construction - SAS

Exercises

1. Which of the two triangles you constructed are acute?
2. Which are obtuse?
3. Which are right?
4. Which are scalene?
5. Which are isosceles?
6. Which are equilateral?
7. Use protractor and ruler to determine the following for each triangle you constructed.

   Triangle 1

   smallest angle is _____ shortest side _____

   second largest angle is _____ second longest side _____

   largest angle is _____ longest side _____

   Triangle 2

   smallest angle is _____ shortest side _____

   second largest angle is _____ second longest side _____

   largest angle is _____ longest side _____

   Triangle 3

   smallest angle is _____ shortest side _____

   second largest angle is _____ second longest side _____

   largest angle is _____ longest side _____

   What do you conclude?

8. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another, are the triangles congruent?

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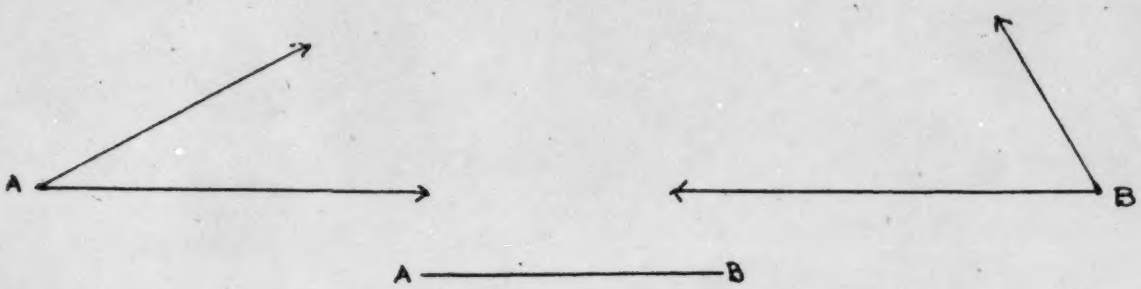
Triangle Construction - ASA

Activity V

Given: Two angles and the included side (this means the side is between the angles or endpoints of the side are the vertices of the angles).



Construct: ΔABC

Compare your triangle to several others constructed from the same given.

Conclusion?

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Triangle Construction - ASA

Exercises

1. If two angles and the included side of one triangle are congruent to two angles and included side of another triangle, are the triangles congruent?

2. Is the triangle you constructed scalene, isosceles or equilateral? acute, right, or obtuse?

3. We have seen triangles constructed by SSS, SAS and ASA. Name the triangle construction you would use if the following measures were given. Name the triangle constructed.

   a. XY, YZ, ZX

   b. $m\angle A$, $m\angle C$, AC

   c. DE, EF, $m\angle E$

   d. GH, $m\angle H$, HI

   e. WX, $m\angle W$, $m\angle X$

   f. AX, XC, CA

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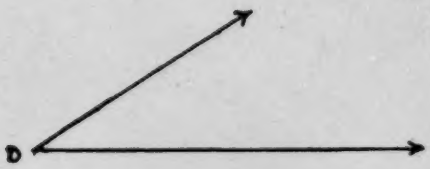
Triangle Construction - AAS

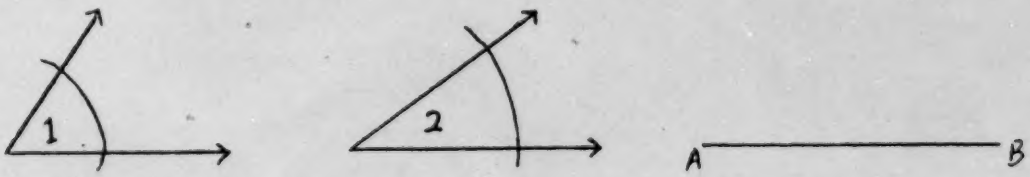
Activity VI

Given: an angle.

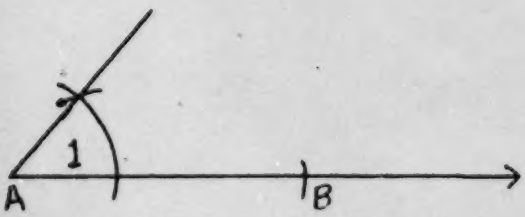


Construct: An angle supplementary to the given angle.

Given: Two angles and a non-included side.



Construct: a triangle.



You may find this construction more difficult. If we construct $\angle 1$ with vertex at A, then we can't have $\angle 2$ with vertex at B since that would make $\overline{AB}$ included.

Where do we construct $\angle 2$?

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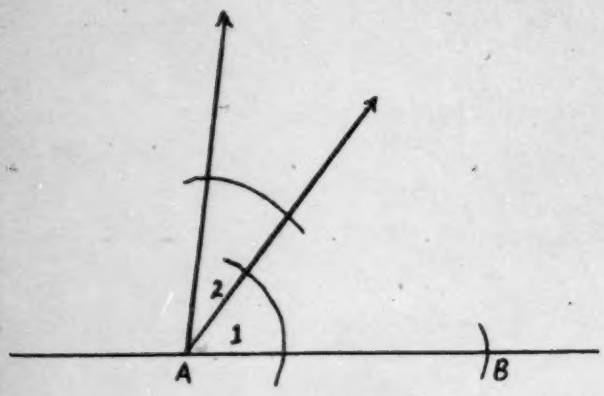
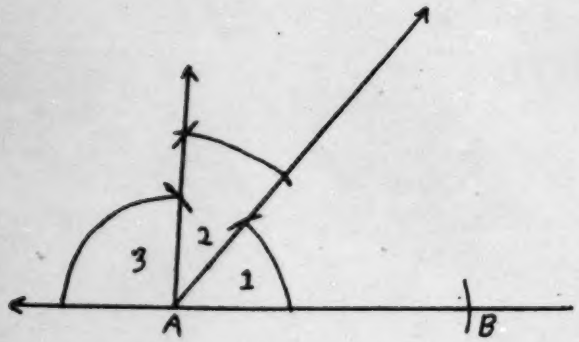


Since $\angle 2$ doesn't have vertex B, lets find the angle that would. We know total measure of the angles of a triangle is 180°, so $m\angle 1 + m\angle 2 + m\angle B = 180^\circ$. Find $m\angle 1 + m\angle 2$.

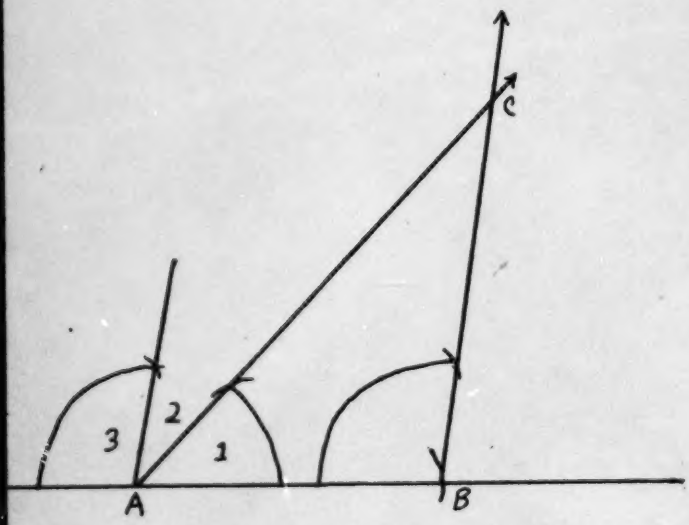


$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

therefore $m\angle B = m\angle 3$.

Construct $\angle B \cong \angle 3$.

LC $\cong$?



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1. Given: two angles of a triangle and a non-included side. Let $\angle A$ and $\angle B$ be any acute angles such that $\angle A \cong \angle B$. Let the non-included side be 5 cm (draw the given).

Construct: ΔABC

Would all other triangles constructed from the same given be congruent to this one?

2. Given: $m\angle A = 45^\circ$, $m\angle B = 55^\circ$, $AC = 6.5$ cm

Construct: ΔABC

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3. Given: $m\angle B = 40°$, $m\angle C = 85°$, $BC = 5cm$

Construct: ΔABC

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Triangle Construction - AAS

Exercises

1. Name the triangle construction you would use if the following measures were given.

   a. AB, BC, CA

   b. $m\angle X$, $m\angle Y$, XY

   c. $m\angle D$, $m\angle E$, DF

   d. $m\angle Y$, XY, ZY

   e. $m\angle G$, $m\angle H$, GI

   f. DE, $m\angle D$, $m\angle E$

2. We have investigated triangle construction when given SSS, SAS, ASA, AAS. Can you think of any other possibilities?

3. If two angles and a non-included side of one triangle are congruent to two angles and a non-included side of another triangle, are the triangles congruent?

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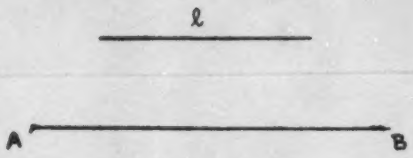
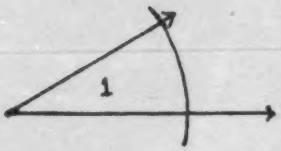
Triangle Construction - SSA

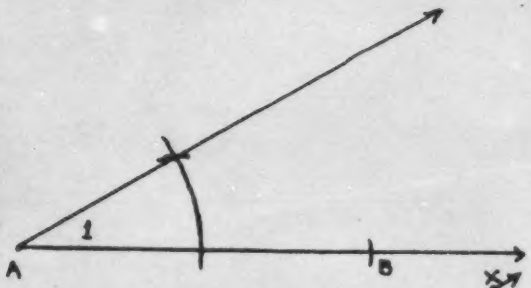
Activity VII

Given: Two sides and a non-included angle.

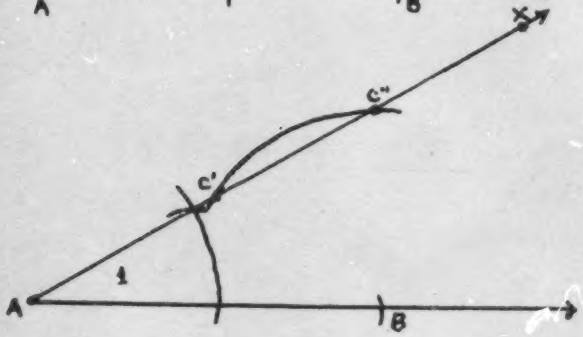




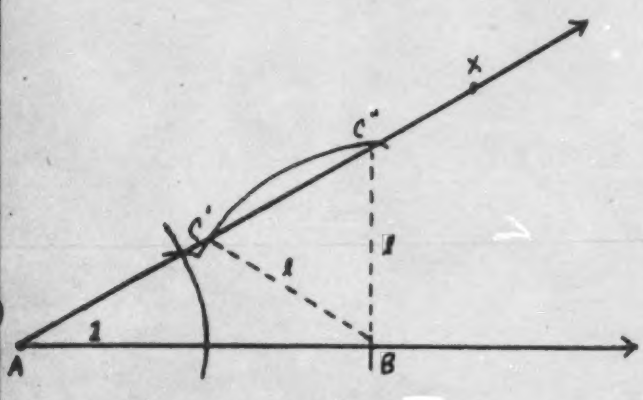
Construct: a triangle.



1. Construct $\overline{AB}$ and $\angle 1$ with vertex at A. Where do we construct side ℓ? It cannot have endpoint at A since that would be construction by SAS.



2. With radius measure of ℓ and center at B construct an arc. Note the arc intersects $\overrightarrow{AX}$ twice: at C' and C".



. It's possible to construct two different triangles from the given: ΔABC' and ABC".

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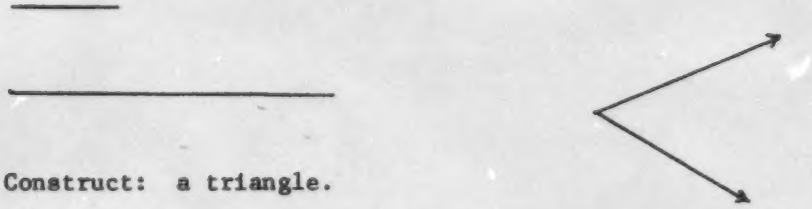
1. Given: Two sides and a non-included angle (use the same given as the last construction).

Construct: A triangle with $\angle 1$ at vertex B.

Did changing the vertex of $\angle 1$ make any changes in the resulting triangles?

2. Given: Two sides and a non-included angle.



Construct: a triangle.

Conclusion?

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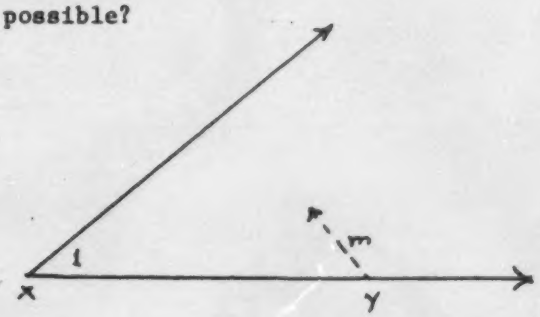
Triangle Construction - SSA

Exercises

1. If two sides and a non-included angle of one triangle are congruent to two sides and the non-included angle of another, are the triangles congruent?

2. Suppose you are constructing a triangle by SSA given $\overline{XY}$, $\angle 1$ and m. Use a ruler to estimate the length of m that would make only one triangle possible. What length of m would make no triangle possible? two triangles possible?



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Triangle Construction - AAA

Activity VIII

1. Given: Three angles (draw the given).

Construct: A triangle.

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Triangle Construction - AAA

Exercises

1. Could a triangle be constructed from any three given angles?

2. Could a triangle be constructed given $m\angle A = 40°$, $m\angle B = 80°$?

3. If three angles of one triangle are congruent respectively to three angles of another, are the triangles congruent?

4. Suppose ΔABC and ΔXYZ are constructed so that $m\angle A = m\angle Y = 72°$, $m\angle B = m\angle Z = 21°$, $CA = XY = 4"$. By which construction do we know these triangles are congruent?

5. By what construction do you know $\Delta ABC \cong \Delta XYZ$?
   a. Given $AB = XY$, $AC = XZ$, $BC = YZ$
   b. Given $m\angle C = m\angle Z$, $m\angle B = m\angle Y$, $AC = XZ$
   c. Given $m\angle C = m\angle Z$, $m\angle B = m\angle Y$, $CB = ZY$
   d. Given $\angle C$, $\angle Z$ are right $\angle s$, $m\angle B = m\angle Y$, $BC = YZ$
   e. Given $m\angle C = m\angle Z$, $AC = XZ$, $BC = YZ$
   f. Given $m\angle A = m\angle X$, $m\angle C = m\angle Z$, $BC = YZ$
   g. Given $\angle C$ and $\angle Z$ are right angles, $AC = XZ$, $m\angle A = m\angle X$

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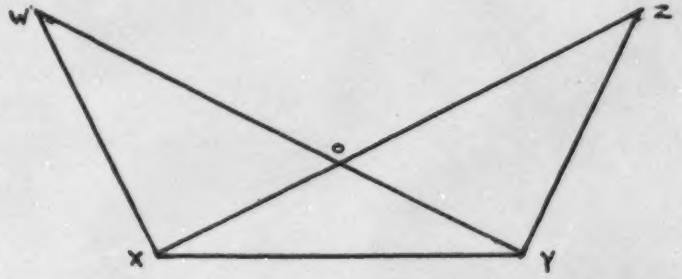
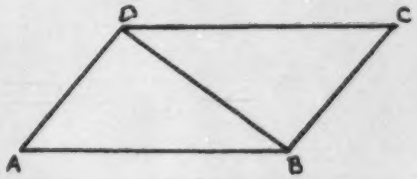
6. Suppose this drawing were constructed so that $\angle WYX \cong \angle ZXY$ and $\overline{WY} \cong \overline{ZX}$.

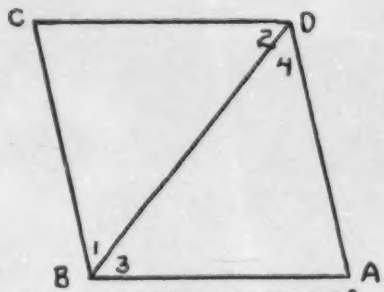
Is $\angle WXY \cong \angle ZYX$?



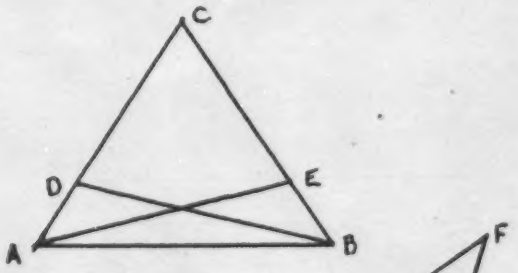
7. Suppose $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$. By what construction are these triangles congruent? Show the vertex correspondence that is a congruent? How do you know $\angle A \cong \angle C$?



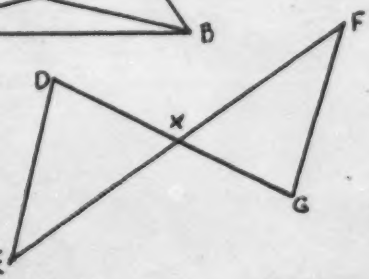
8. Suppose $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$. By what construction are the triangles congruent? Show the vertex correspondence. How do you know $\overline{AD} \cong \overline{CB}$?



9. $\overline{AC} \cong \overline{BC}$, $\overline{DC} \cong \overline{EC}$. Which triangles can you prove congruent? By what construction?



10. Suppose $LD \cong LG$, $DX \cong XG$. By what construction can you prove these triangles congruent?



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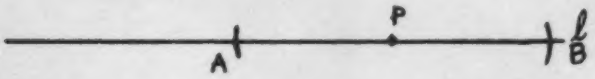
Triangle Construction - HL

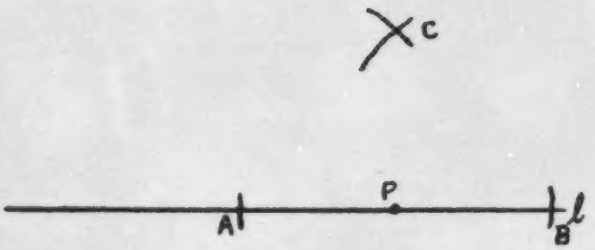
Activity IX

Given: line segment ℓ containing point P.

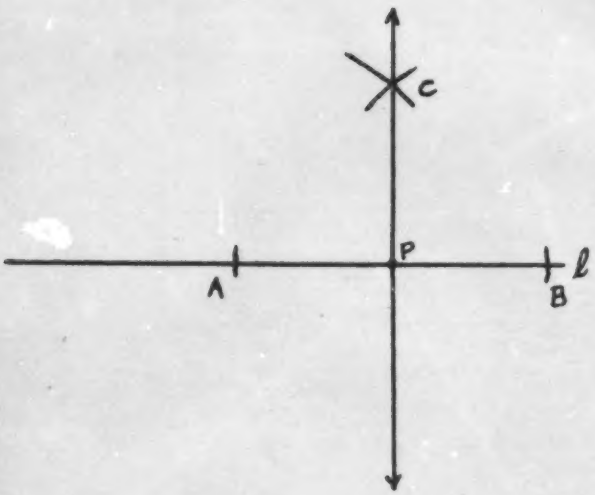
Construct: a line perpendicular to ℓ at P.



1. With P as center and any convenient radius construct arcs that intersect ℓ. Call the points of intersection A. and B.



2. With radius greater than AP and center first at A, then at B, contruct intersecting arcs. Call the point of intersection C.



3. Draw $\overleftrightarrow{PC}$.

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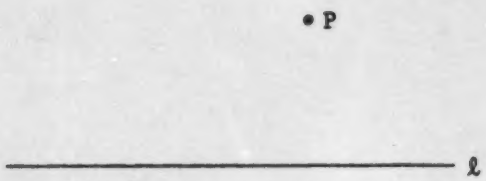
How do you know $\overleftrightarrow{PC} \perp \ell$?

$\overline{AP} \cong \overline{BP}$, $\overline{AC} \cong \overline{BC}$	by construction
$\overline{PC} \cong \overline{PC}$	why?
$\triangle ACP \cong \triangle BCP$	by what construction?
$\angle APC \cong \angle BPC$	why?
$PC \perp \ell$	why?

1. Given: line segment ℓ, point P not on ℓ.

Construct: a line perpendicular to ℓ through p. (You do it.) The construction is left to you.



1. With P as center and a convenient radius construct an arc that intersects ℓ at two points call the points A and B.

2. With radius greater than 1/2 AB and center first at A and then at B construct intersecting arcs. Call the point of intersection C.

3. Draw $\overleftrightarrow{PC}$.

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Triangle Construction - HL

Activity Card

How do you know $\overleftrightarrow{PC} \perp \ell$?

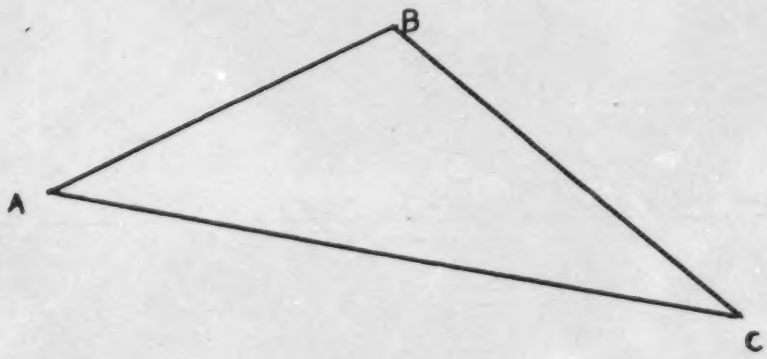
$\overline{PA} \cong \overline{PB}$, $\overline{AX} \cong \overline{BX}$	why?
$\overline{PX} \cong \overline{PX}$	why?
$\Delta PAX \cong \Delta PBX$	why?
$\angle APX \cong \angle BPX$	why?

Call the point where PC and ℓ intersect point Y.

$\overline{PY} \cong \overline{PY}$	why?
$\Delta APY \cong \Delta BPY$	why?
$\angle PYA \cong \angle PYB$	why?
$\overleftrightarrow{PC} \perp \ell$	why?

2. Given: ΔABC

Construct: a line perpendicular to $\overline{AC}$ from point B.



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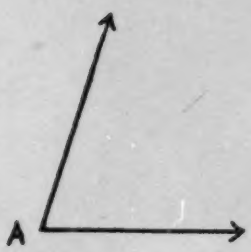
Triangle Construction - HL

Activity Card

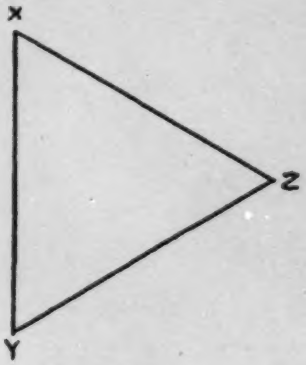
3. Given: Any acute angle.



Construct: an angle complementary to the given angle.

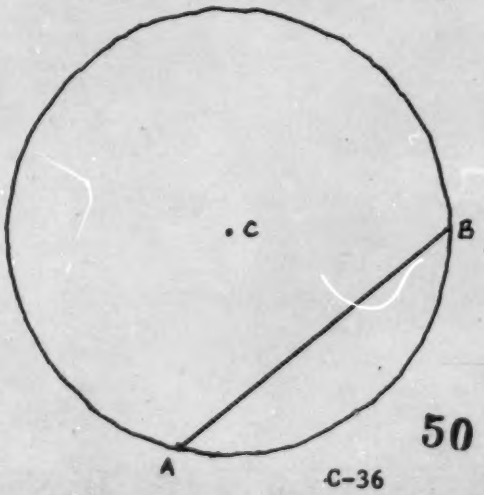
4. Given: ΔXYZ

Construct: A line perpendicular to $\overline{XY}$ through Y.



5. Given: $\odot C$ with chord $\overline{AB}$.

Construct a radius perpendicular to $\overline{AB}$ through C.



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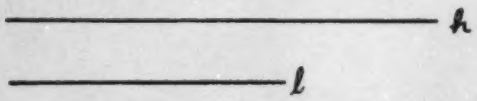
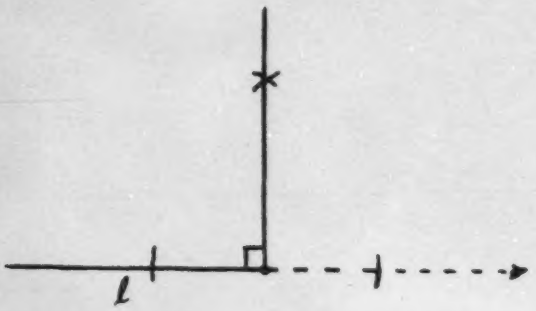
Definition: A right triangle is a triangle that contains one right angle. The side opposite the right angle is called the hypotenuse. The other two sides are called the legs of the right triangle.

Given: the hypotenuse h and a leg ℓ of a right triangle.

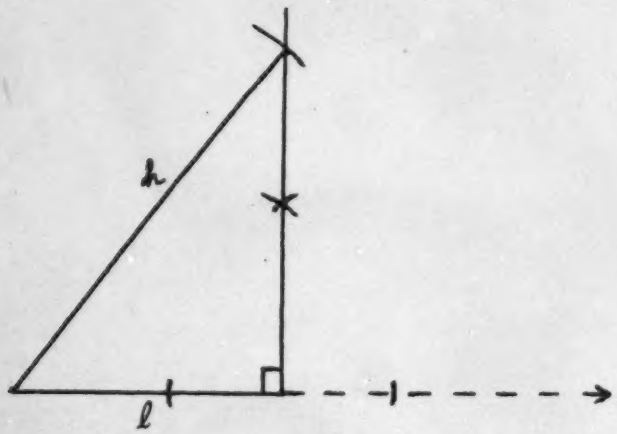


Construct: a right triangle.



1. Construct a line perpendicular to ℓ through an endpoint of ℓ.

2. With center at other endpoint of ℓ and radius h construct an arc intersecting the perpendicular. Draw A.



Compare your triangle with any other constructed from the same given.

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, are the triangles congruent?

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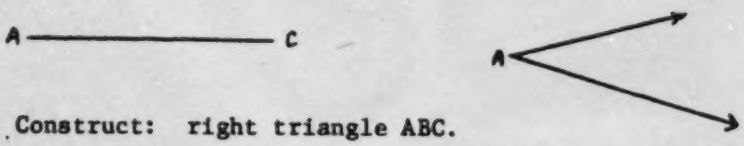
6. Given: leg $\overline{AC}$, $\angle A$ an acute angle.



Construct: right triangle ABC.

7. Given: AC = 3", BC = 2" (Draw the given).

Construct: right triangle ABC with right angle C.

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8. Given: $m\angle A = 35°$, $AB = 4$ cm (Draw the given).

Construct: right triangle ABC with right angle C.

9. Given: measure of a leg is 3.5 cm, measure of hypotenuse is 7 cm (Draw the given).

Construct: a right triangle.

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Triangle Construction - HL

Exercises

1. What method of triangle construction did you use on each triangle?

   Triangle 1 -

   Triangle 2 -

   Triangle 3 -

   Triangle 4 -

2. By what construction do you know $\Delta XYZ \cong \Delta ABC$?

   a. Given $\angle C$, $\angle Z$ right angles, $\overline{XY} \cong \overline{AB}$, $\angle X \cong \angle A$

   b. Given $\angle C$, $\angle Z$ right angles, $\overline{YZ} \cong \overline{BC}$, $\overline{ZX} \cong \overline{CA}$

   c. Given $\angle C$, $\angle Z$ right angles, $\overline{XY} \cong \overline{AB}$, $\overline{ZX} \cong \overline{CA}$

   d. Given $\angle C$, $\angle Z$ right angles, $\overline{YZ} \cong \overline{AB}$, $\angle Y \cong \angle B$

   e. Given $\angle C$, $\angle Z$ right angles, $\overline{YZ} \cong \overline{BC}$, $\overline{XY} \cong \overline{AB}$

   f. Given $\overline{XY} \cong \overline{AB}$, $\overline{YZ} \cong \overline{BC}$, $\overline{XZ} \cong \overline{AC}$

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Arguing for Congruence

Information Sheet

This section shall be used to return you to your favorite activity; writing proofs. You already have much information useful in forming proofs. For example, you know if two triangles are congruent, then their corresponding parts are congruent. You know five ways to show that two triangles are congruent: SSS, SAS, ASA, AAS, HL. We shall add to this some theorems about isosceles triangles and perpendicular bisectors, provide proofs of these for you to use as samples and allow you to proceed.

Definition: If two sides of a triangle are congruent, the triangle is called isosceles. The remaining side is called the base. The two angles sharing the base as a side are called base angles. The third angle is called the vertex angle.

Definition: An altitude of a triangle is the perpendicular segment from any vertex to a line containing the opposite side.

Definition: A median of a triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite sides.

Definition: An angle bisector of a triangle is a segment whose endpoints are the points of intersection of the triangle and a ray that bisects an angle of the triangle.

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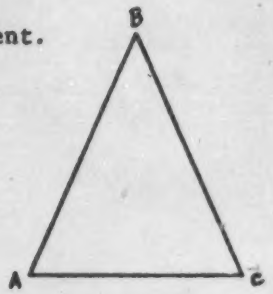
Theorem: Base angles of an isosceles triangle are congruent.



Restatement: Given ΔABC with $\overline{AB} \cong \overline{BC}$

Prove $\angle A \cong \angle C$

Proof: Consider the correspondence between ΔABC and itself given by $\Delta ABC \leftrightarrow \Delta CBA$.

Since $\overline{AB} \cong \overline{BC}$ and $\overline{AC} \cong \overline{CA}$, $\Delta ABC \cong \Delta CBA$ by SSS.

Hence $\angle A \cong \angle C$ since they are corresponding parts of congruent triangles.

Theorem: If two angles of a triangle are congruent, the sides opposite those angles are congruent.

Restatement: Given ΔABC with $\angle A \cong \angle C$

Prove $\overline{AB} \cong \overline{BC}$

Proof: Consider the correspondence $\Delta ABC \leftrightarrow \Delta CBA$

Since $\angle A \cong \angle C$ and $\overline{AC} \cong \overline{AC}$

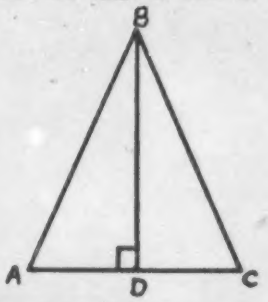
$\Delta ABC = \Delta CBA$ by ASA

and $\overline{BC} \cong \overline{AB}$ since they are corresponding sides.

Theorem: An altitude to the vertex angle of an isosceles triangle is also a median and an angle bisector.

Restatement: Given: ΔABC with $\overline{AB} \cong \overline{BC}$ and $\overline{BD}$ such that $\overline{BD} \perp \overline{AC}$ and D is between A and C.

Prove: $AD \cong DC$ and $\angle ABC \cong \angle CBD$.



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Proof: $\overline{AB} \cong \overline{BC}$ and $\overline{BD} \cong \overline{BD}$

implies that $\triangle ABD = \triangle CBD$ by HL which

implies that $\overline{AD} = \overline{DC}$ and

$\angle ABC = \angle CBD$ by corresponding parts of congruent triangles.

Theorem: The perpendicular bisector of a segment is the set of all points equidistant from the endpoints of that segment.

Restatement: Given: Segment $\overline{AB}$ and line ℓ which is the perpendicular bisector of $\overline{AB}$ at M.

Prove: If C is on ℓ then CA = CB and if CA = CB, then C is on ℓ.

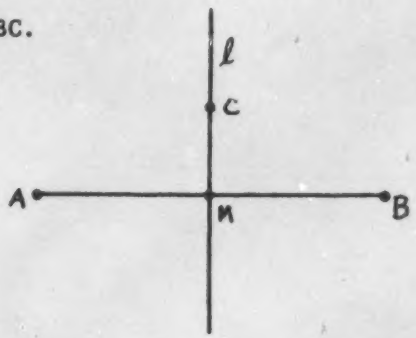
Proof: If C is on ℓ and $\overline{AB}$ then C = M and AC = BC.

If not, $\overline{AM} \cong \overline{MB}$, $\overline{CM} \cong \overline{CM}$ and $\angle CMA \cong \angle CMB$

$\Rightarrow \triangle CMA \cong \triangle CMB$ by SAS

$\Rightarrow \overline{AC} \cong \overline{BC}$

$\Rightarrow AC = BC$.



If CA = CB and $\overline{CM}$ is constructed $\triangle CMB = \triangle CMA$ by SSS and

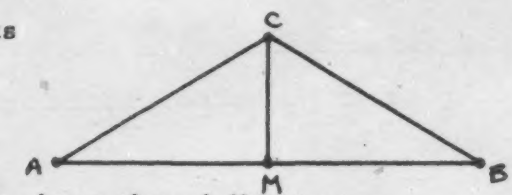
$\Rightarrow \angle CMB \cong \angle CMA$ by corresponding parts

$\Rightarrow \angle CMB$ and $\angle CMA$ are right angles

$\Rightarrow \overleftrightarrow{CM} \perp \overline{AB}$

$\Rightarrow \overleftrightarrow{CM} = \ell$ since only one line may be drawn through M perpendicular to $\overline{AB}$.

$\Rightarrow$ C is on ℓ.



For your proofs it is important that you include three things:

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1. A model or diagram that may be used to visualize the statement to be proven.
2. The statement to be proven should be written down using the names for points and lines used in the model.
3. A chain of reasoning similar to those given in the proofs of the four theorems here should be included.

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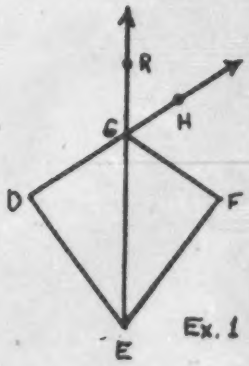
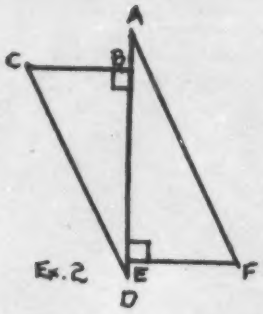
Arguing for Congruence

Exercises



Ex. 1

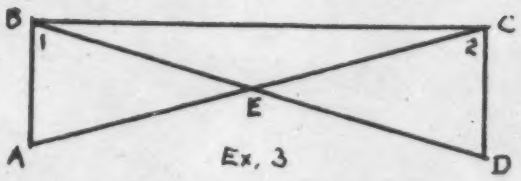
1. If $\angle RGH \cong EGF$ and GD = GF, prove that $\angle D \cong \angle F$.



Ex. 2

2. Given: AB = DE, FE = CB, $\overline{FE} \perp \overline{AD}$, $\overline{CB} \perp \overline{AD}$

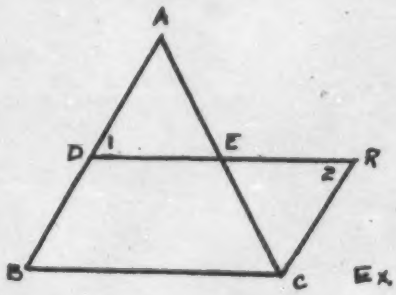
   Prove: $\angle F \cong \angle C$



Ex. 3

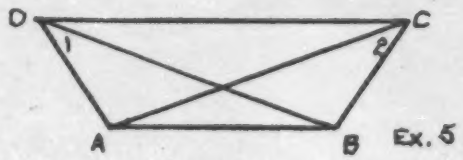
3. Given: $\overline{BE} \cong \overline{EC}$, $\angle 1 \cong \angle 2$, $\overline{AE} \cong \overline{ED}$

   Prove: AB ≅ CD



Ex. 4

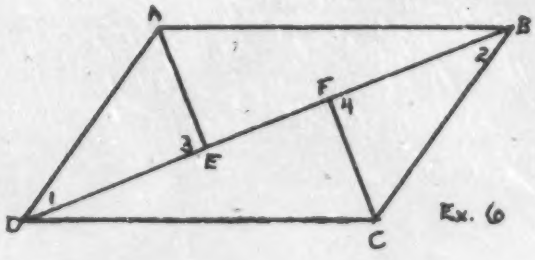
4. If $\overline{AC}$ bisects $\overline{DR}$ and $\angle 1 \cong \angle 2$, prove that $\overline{DR}$ bisects $\overline{AC}$.



Ex. 5

5. Given: AC = BD, AD = BC

   Prove: $\angle 1 \cong \angle 2$



Ex. 6

6. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

   DF = BE

   Prove: AB = DC

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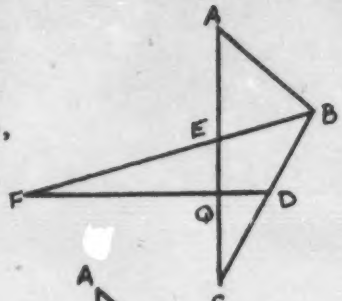
7. Given: $\angle FDB \cong \angle CBA$, $BD = AB$, $\angle BAE \cong \angle DBF$

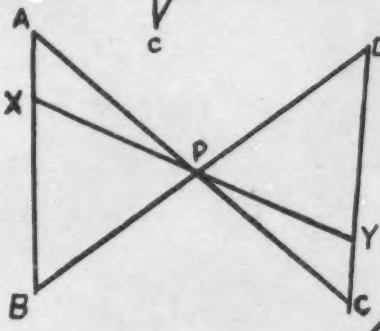
Prove: $FD = BC$



Ex. 7

8. Given: $AP = PC$ and $BP = PD$

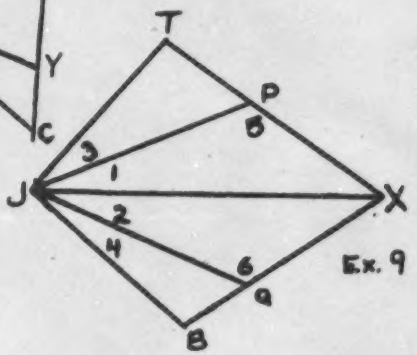
Prove: $AX = CY$



Ex. 8

9. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $JT = JB$

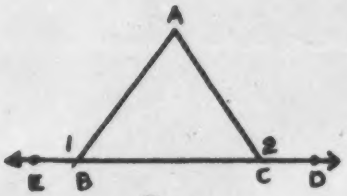
Prove: $\angle 5 \cong \angle 6$



Ex. 9

10. Given: $\overline{AB} \cong \overline{AC}$

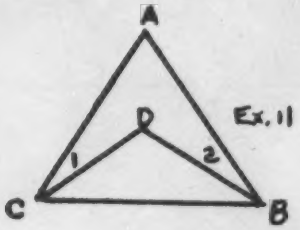
Prove: $\angle 1 \cong \angle 2$



Ex. 10

11. Given: $AC = AB$ and $DC = DB$

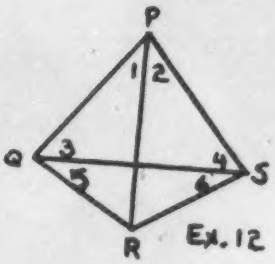
Prove: $\angle 1 \cong \angle 2$



Ex. 11

12. Given: $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$

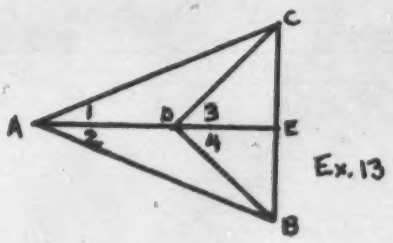
Prove: $\angle 1 \cong \angle 2$



Ex. 12

13. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

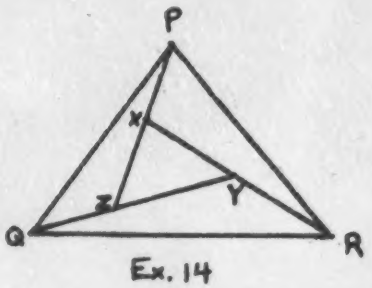
Prove: $\overline{AE} \perp \overline{CB}$



Ex. 13

14. Given: ΔXYZ is equilateral and $XP = ZQ = YR$

Prove: ΔPQR is equilateral.



Ex. 14

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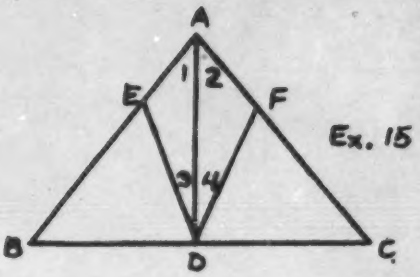
15. Given: $\overrightarrow{AD}$ bisects $\angle BAC$, $\angle 3 \cong \angle 4$

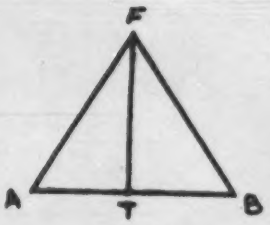
Prove: $\overline{DE} \cong \overline{DF}$



Ex. 15

16. Given: $\overline{FT}$ is a median, $\overline{FT} \perp \overline{AB}$

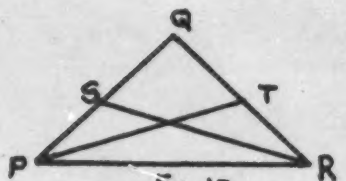
Prove: FA = FB



Ex. 16

17. Given: $\overline{PT}$ and $\overline{RS}$ are medians of ΔPQR. $\overline{QP} \cong \overline{QR}$

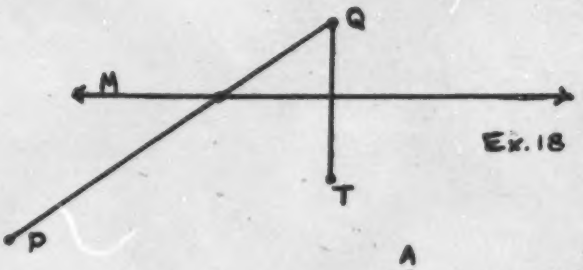
Prove: $\overline{PT} \cong \overline{RS}$



Ex. 17

18. Given: Line m is the perpendicular bisector of $\overline{QT}$

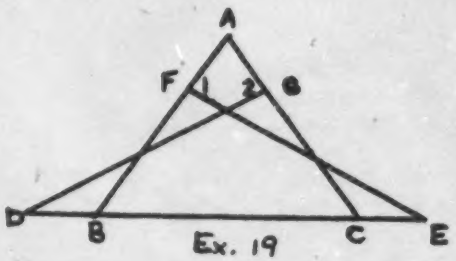
Prove: PQ = PR + RT



Ex. 18

19. Given: AB = AC, AF = AG, $\angle 1 \cong \angle 2$

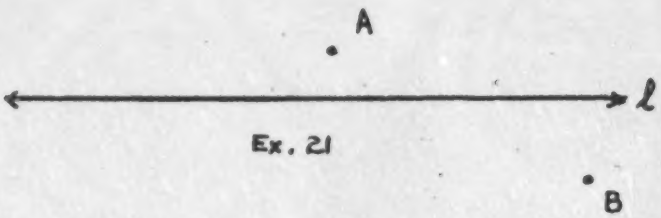
Prove: $\angle D \cong \angle E$



Ex. 19

20. Prove that if two medians of a triangle are perpendicular to their repective sides, then the triangle is equilateral.

21. Using a compass and straight edge locate the point on ℓ which is equidistant from A and B.



Ex. 21

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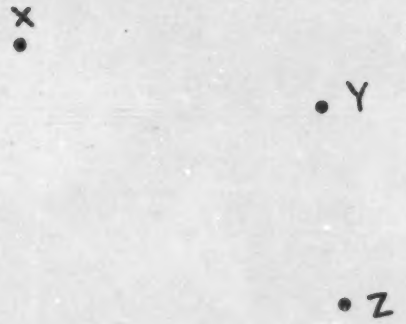
22. Using a compass and straight edge, locate a point equidistant from X, Y, and Z.



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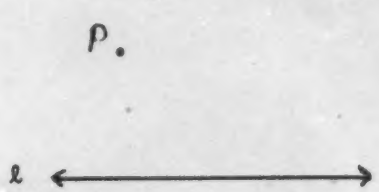
Parallel Lines - Construction

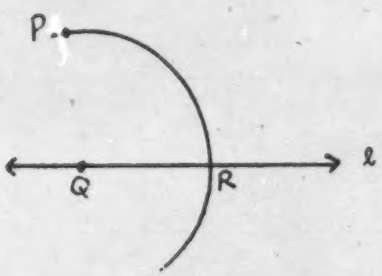
Activity X

Construct a parallel to a given line through a given point.

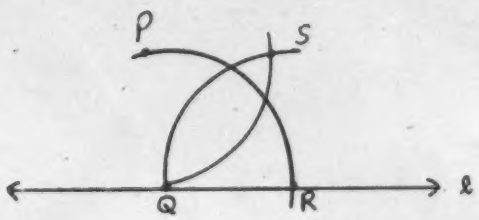


Given: Line ℓ and point P not on ℓ.

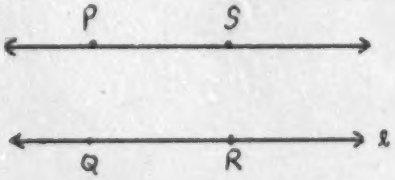
Step 1: Choose any point Q on ℓ; construct an arc of radius PQ and center Q which intersects ℓ at R.



Step 2: With P and R as centers and PQ as the radius, construct arcs which intersect each other at S.



Step 3: $\overleftrightarrow{PS}$ is the required line parallel to ℓ.



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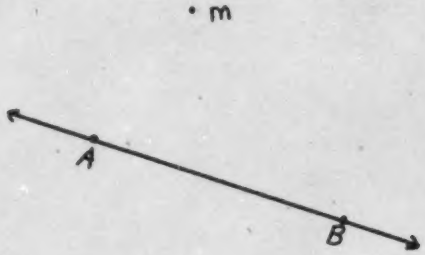
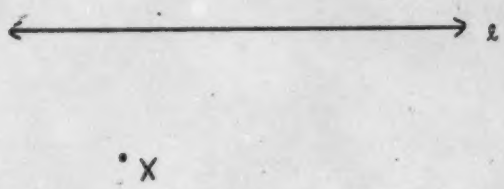
Parallel Lines - Construction

Exercises

1. Construct a parallel to $\overleftrightarrow{AB}$ through M.



2. Construct a parallel to ℓ through X.



3. Describe a method of constructing a line parallel to a given line through a point not on the line, by using
   a) corresponding angles.
   b) alternate interior angles.

4. Construct a parallelogram, given the lengths of two adjacent sides and the length of a diagonal. What restrictions must be placed on the length of the diagonal?

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5. Given segments $\overline{AB}$ and $\overline{CD}$ and any angle, $\angle Y$. Construct a parallelogram PQRS such that $AB = PQ$, $CD = QR$, and $\angle Q \cong \angle Y$.

6. Given a line ℓ and a segment $\overline{AB}$ of length 1. Construct two lines that are parallel to ℓ and are at a distance 1 from ℓ.

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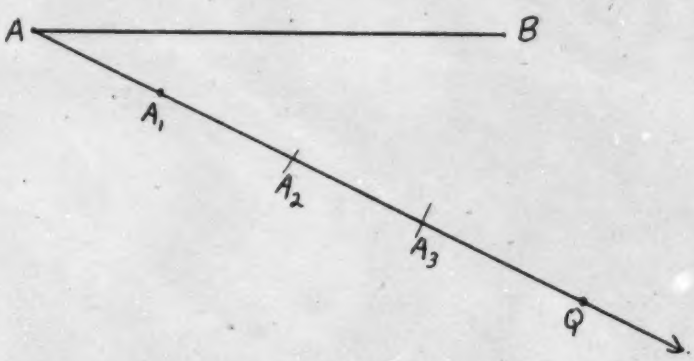
Subdivide a Segment

Activity XI

Now subdivide a given segment into three congruent segments.

Given: segment $\overline{AB}$. Subdivide $\overline{AB}$ into three congruent segments.

Step 1. Choose any point Q, not on $\overline{AB}$ and draw $\overrightarrow{AQ}$.

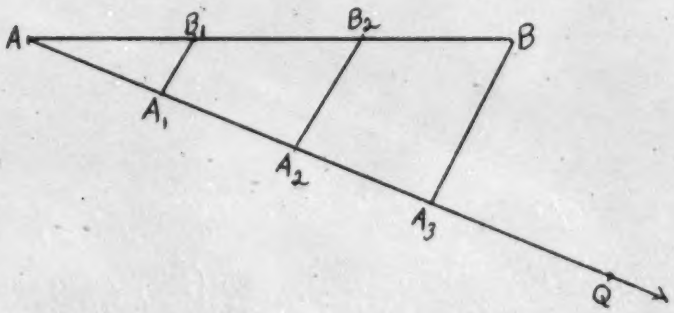


Step 2. Choose a point A_1 on AQ and find A_2 and A_3 such that

$AA_1 = A_1A_2 = A_2A_3$. (See figure above.)

Step 3. Draw $\overline{A_3B}$. Construct lines parallel to $\overline{A_3B}$ through A_1 and A_2.

(See Parallel Line Construction Exercises, #3a).



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Let B_1 and B_2 be the points of intersection of these parallels with $\overline{AB}$. Then, $\overline{AB}_1 \cong \overline{B_1B_2} \cong \overline{B_2B}$.

You could use the same procedure to subdivide a given segment into n (n an integer) subdivisions.

An alternate method will be presented in the unit on similarity.

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Subdivide a Segment

Exercises

1. Bisect a given segment $\overline{RS}$.

2. Draw a segment $\overline{AB}$. Then construct an equilateral triangle whose perimeter is AB.

3. Draw any segment $\overline{AB}$. Construct segment $\overline{PQ}$ such that PQ = 2/3 AB.

4. Given segment $\overline{AB}$. Draw ray $\overrightarrow{AC}$. Now construct ray $\overrightarrow{BD} // \overrightarrow{AC}$. Mark off four congruent segments on $\overrightarrow{BD}$ (X_1, X_2, ...) and on $\overrightarrow{AC}$ (Y_1, Y_2, ...). Draw AX_4, Y_1X_3, Y_2X_2, Y_3X_1, and Y_4B. Is this method of subdividing a segment correct? Why or why not?

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5. Draw any segment $\overline{AB}$. Construct a square whose perimeter is AB.

6. Draw any segment $\overline{PR}$. Construct a rectangle whose length is twice the width and perimeter is $\overline{PR}$.

7. Draw three segments $\overline{AB}$, $\overline{DE}$, and $\overline{PQ}$ such that $AB < DE < PQ$. Find a point M on PQ such that $\frac{PM}{MQ} = \frac{AB}{DE}$.

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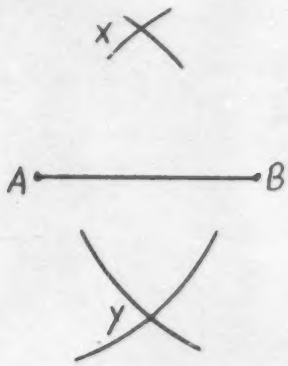
Altitudes, Medians and Angle Bisectors

Activity XII

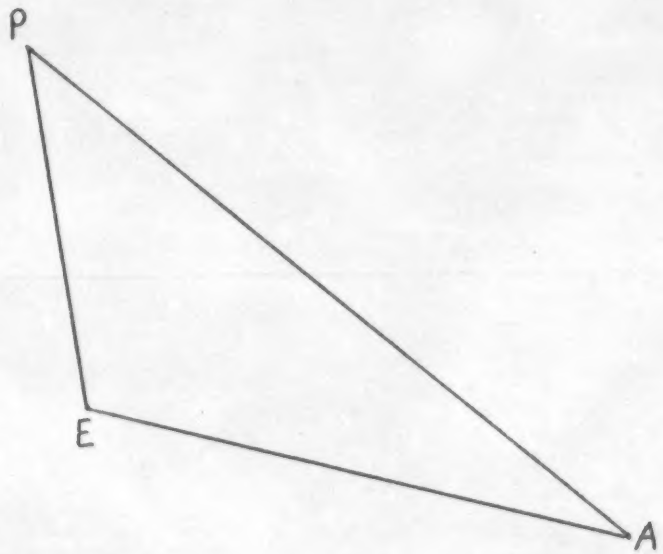
Construct the perpendicular bisector of segment $\overline{AB}$.



Solution

Find points X and Y such that AX = BX and AY = BY. It follows that $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

Construct a perpendicular bisector to each side of ΔEAP.



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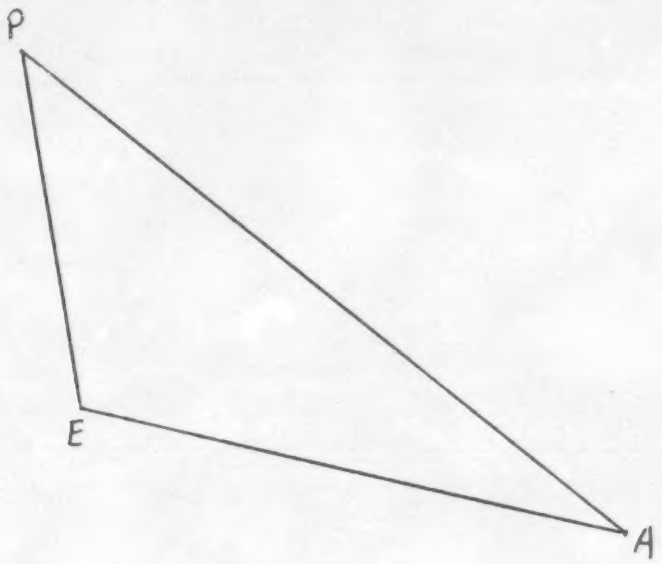
Questions

1. Are the perpendicular bisectors of the sides concurrent (having a common point of intersection?)
2. Call the point of intersection H. How do the lengths HE, HA and HP compare?

Construct all three medians in ΔEPA.



3. Are the medians concurrent (having a common point of intersection?)
4. Call the point of intersection G. If the medians are named $\overline{BA}$, $\overline{RE}$, and $\overline{PI}$, compare the following pairs of lengths:

   EG, GR; AG, BG and GI, PG.

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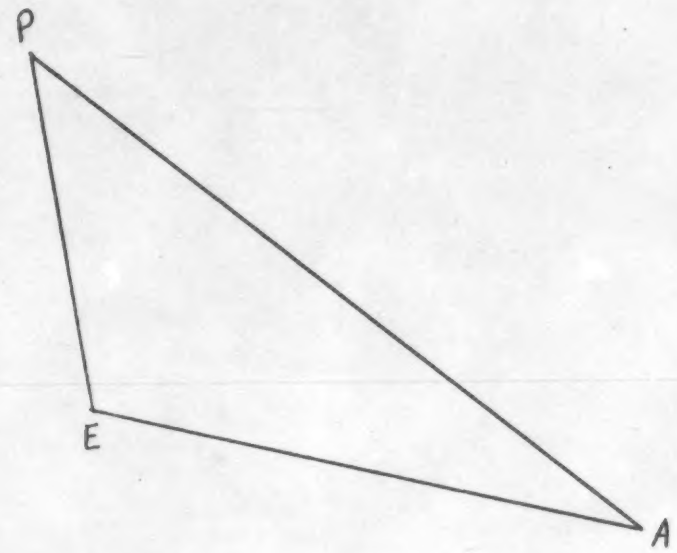
Construct all three altitudes in triangle PAE.

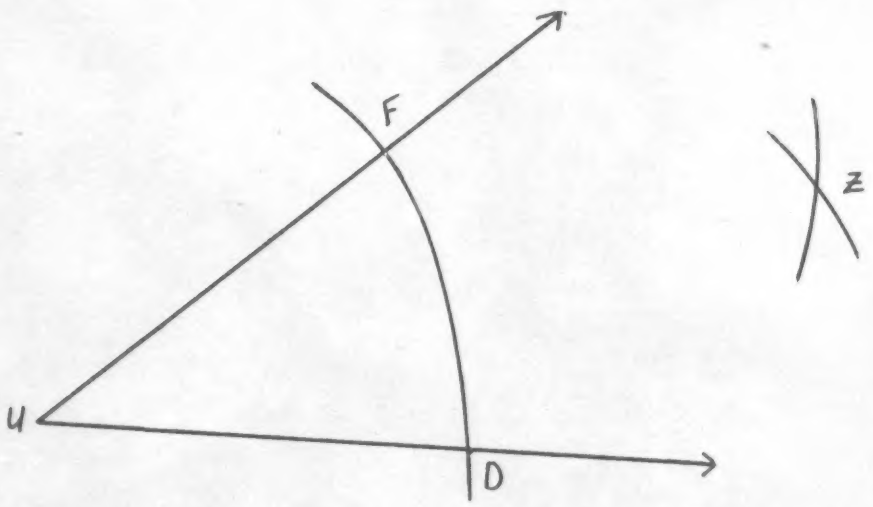


Questions

5. Are the altitudes concurrent?

6. Are the lines containing the altitudes concurrent?

Construct the angle bisector of ∠U.



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Solution

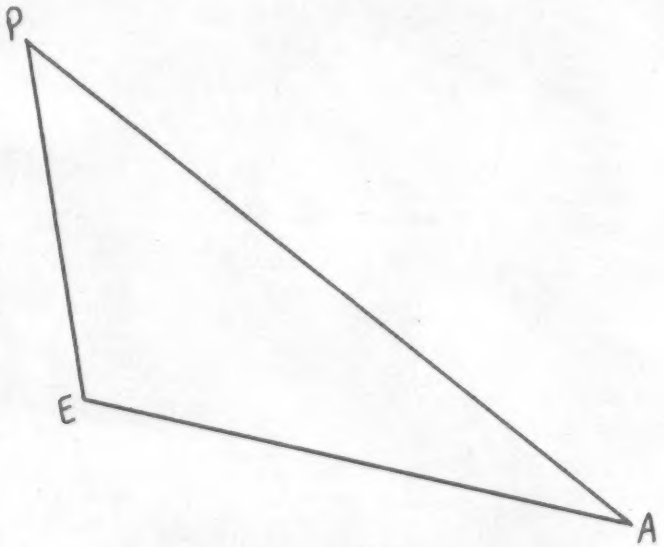
Determine points F and D on the sides of angle U such that FU = DU. Determine point Z in the interior of angle Z such that FZ = DZ. It follows that $\overrightarrow{UZ}$ is the angle bisector.

Question

7. Why does it follow that $\overrightarrow{UZ}$ bisects $\angle FUD$?

Construct a 45° angle.

Construct the angle bisectors of $\triangle APE$.



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Question

8. Are the angle bisectors concurrent?

Let T be the point of intersection of the angle bisectors. Construct segments from T to each of the sides of the triangle such that these segments are perpendicular to their respective sides. Call these segments TM, TN and OT.

Question

9. How do the lengths of TM, TN and OT compare?

In this section you have found four points which are defined ΔEPA and its auxiliary parts. All of these points have names to identify them which are in some way descriptive of the point. The names are as follows:

Point of concurrency of	Name
Perpendicular bisectors	Circumcenter
Medians	Centroid (center of gravity)
Altitudes	Orthocenter
Angle bisectors	Incenter

We mention this because it raises many interesting questions, some of which are listed here as optional questions for investigation.

Questions

10. What significance does each of the names possess?

11. Could you balance a cutout of ΔPAE on a pencil point at its centroid?

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12. Does any relationship exist between these points?

13. Do these points have the same significance for every triangle?

14. Do these points have special properties for special triangles?

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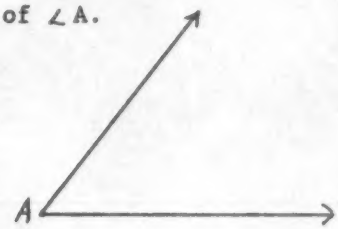
Altitudes, Medians and Angle Bisectors

Exercises

1. Construct an angle of measure 30.
2. Construct an angle of measure 75.
3. Construct an angle of measure 135.
4. Given a segment $\overline{AB}$ and an acute angle $\angle C$, construct a right triangle so that $\overline{AB}$ is the altitude drawn to the hypotenuse and $m\angle C$ is the measure of one of the acute angles.
5. Construct an equilateral triangle with $\overline{GU}$ as a median.

6. Construct the supplement of $\angle A$.



7. Construct a triangle given
   a) two sides and a median to one of them.
   b) two sides and the altitude to one of them.
   c) an angle, the bisector of the given angle and a side adjacent to the given angle.

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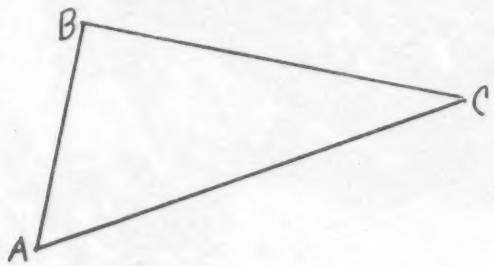
Triangle Inequalities

Activity XIII

We have shown that: if two angles of a triangle are congruent, the sides opposite those angles are also congruent. Write the contrapositive of this statement.

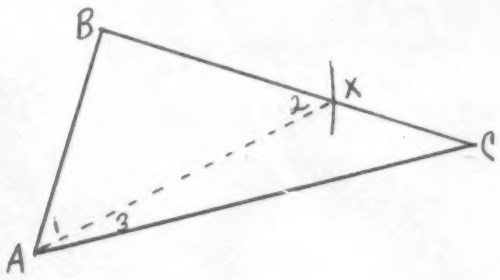
Suppose ΔABC has two sides not congruent, $\overline{AB} \neq \overline{BC}$.



Then by your contrapositive $\angle A \neq \angle C$.

Which is true, $m\angle A < m\angle B$ or $m\angle A > m\angle B$?

Let $AB < BC$. Then on $\overline{BC}$, locate a point X such that $AB = BX$. Draw $\overline{AX}$.



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Now we have an isosceles ΔABX and $m\angle 1 = m\angle 2$. Also, $m\angle 2 = m\angle 3 + m\angle C$. Why?

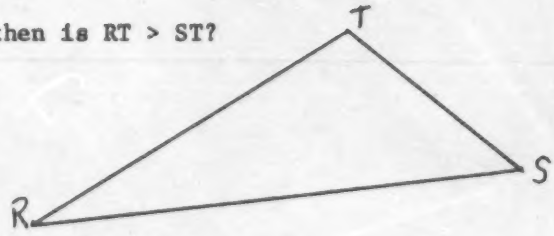
This implies $m\angle 2 > m\angle C$ and substituting: $m\angle 1 > m\angle C$. We can easily see $m\angle A > m\angle 1$, so putting the last two statements together, $m\angle A > m\angle 1 > m\angle C$. We can conclude $m\angle A > m\angle C$.

It appears that: if two sides of a triangle are not congruent, the angles opposite them are not congruent, and that the largest angle is opposite the longest side.

Now write the converse of the contrapositive you wrote.

Is it true?

In ΔRST if $m\angle R < m\angle S$, then is $RT > ST$?



Assume $RT \not> ST$. Then either $RT = ST$ or $RT < ST$.

1. If $RT = ST$, ΔRST is isosceles and $m\angle R = m\angle S$. This contradicts the given fact that $m\angle R < m\angle S$.
2. If $RT < ST$ then by your contrapositive $m\angle R > m\angle S$ this also contradicts the given fact.

The assumption must be false, the converse you wrote is true.

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In constructing triangles by SSS you discovered that you could not construct a triangle out of just any three line segments.

Let two paper strips of different lengths represent two sides of a triangle. Measure and record the lengths.

First side _____

Second side _____

Let the edge of a ruler represent the third side. Experiment until you have determined the longest and shortest possible third side. Record the result.

Third side must be greater than _____ and shorter than _____.

The sum of the lengths of two sides of a triangle must be greater than _____.

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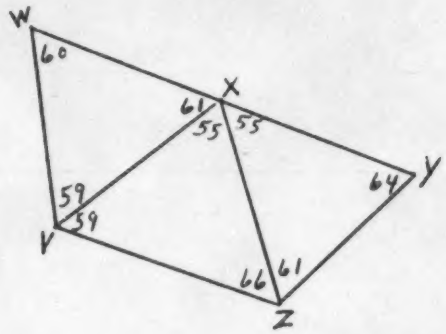
Triangle Inequalities

Exercises

1. Find the longest segment shown.

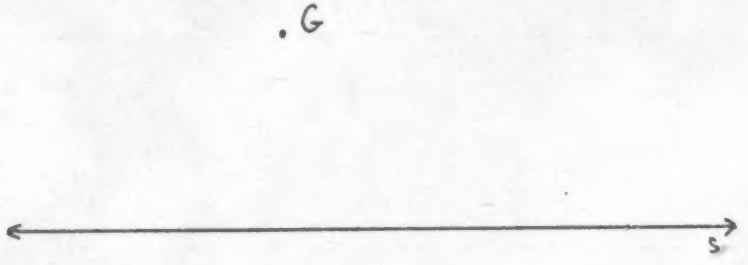


2. Given: ΔCEB

The sides in order from shortest to longest are:	List angles in order from smallest to largest
$\overline{EB}$	_____
$\overline{EC}$	_____
$\overline{CB}$	_____

Conclusion:

3. Let G be a garage and let s be a street. Draw the shortest straight driveway from the garage to the street. Call the point where the driveway intersects the street point A.



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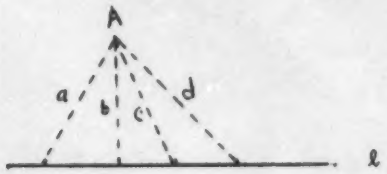
Now suppose there is a tree growing right where you want to put the shortest driveway. (No, you cannot cut it down.) Show an alternate driveway and call the point where it intersects the street point B.

How do you know your first choice was the shortest? Consider ΔGBA. Which is smaller, $m\angle A$ or $m\angle B$? Therefore, which is shorter, $\overline{GA}$ or $\overline{GB}$? We can conclude: the shortest segment from a point to a line must be _____.

4. The distance from a point A to a line ℓ could be measured along many different lines. Along which line segment would you measure from A to ℓ? Give a reason for your answer.



5. Write a definition of the distance from a point to a line.

6. If two sides of a triangle are 2 and 7, the length of the third side must be less than _____ and greater than _____.

7. If two sides of a triangle are x and y with $x > y$, the length of the third side must be less than ____ and greater than ____.

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8. Which of the following could be the lengths of the sides of a triangle?

a) 2, 3, 4

b) 6, 6, 12

c) 6, 6, 13

d) 3, 4, 5

e) x, y, x + y

f) 9, 12, 10

g) x, 2y, x + y

h) 3, 4, 7

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